

The stability of rotating heavy nuclei against fission

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Abstract

A recent paper has reported the observation of the rotational band of ^{254}No for spins up to $I=20$, showing that the compound nucleus was formed and survived fission decay at angular momenta $I \geq 20$. We show that this survival is consistent with the leading effects of angular momentum on the fission barrier height.

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A recent paper [1] has reported the observation of the rotational band of the nucleus ^{254}No for spins up to $I=20$, showing that the compound nucleus was formed and survived fission decay at angular momenta $I \geq 20$.

This finding may appear surprising at first, given the well known instability toward fission of these nuclei and the expected decrease in the fission barrier due to angular momentum [1]. The question behind the surprise is why angular momentum, usually so effective in decreasing the fission barrier and in enhancing the fission to neutron emission branching ratio in lighter nuclei, appears here to be somewhat ineffective. The explanation of this puzzle is not only interesting for this case, but is also even more relevant for the resilience of superheavy nuclei to increasing angular momentum.

Clearly, there is the simple fact that the natural scale for the moment of inertia is $A^{5/3}$ which makes the observed dimensionless angular momenta and rotational energies rather small for $A=254$ compared to what they would be for instance with nuclei of $A \approx 200$. However there is another important reason for the “survival” of the fission barrier, namely, the very small difference in deformation (and thus in the moment of inertia) between the saddle point and the ground state configurations.

To show the leading effects of angular momentum on the barrier height, we use perturbation theory. According to this theory, the energy associated with the perturbation (rotational energy) must be calculated using parameter values (moments of inertia, deformation, etc.) associated with the unperturbed system. This is the standard cranking approximation. Accordingly, the energy can be written as:

$$E(\vec{\epsilon}) = E_0(\vec{\epsilon}) + \frac{I(I+1)\hbar^2}{2\mathcal{J}(\vec{\epsilon})} \quad (1)$$

where $\vec{\epsilon}$ is a generalized deformation vector, $E_0(\vec{\epsilon})$ and $\mathcal{J}(\vec{\epsilon})$ are the potential energy surface and moment of inertia at zero angular momentum.

As shown in Fig. 1, the decrease of the barrier height as a function of angular momentum ($\Delta B(I)$) is

$$\Delta B(I) = B_f(I) - B_f(0) = \frac{\hbar^2}{2} \left(\frac{1}{\mathcal{J}_g} - \frac{1}{\mathcal{J}_s} \right) I(I+1) \quad (2)$$

where \mathcal{J}_g and \mathcal{J}_s are the moments of inertia of the ground state and saddle deformations calculated at $I=0$. This decrease depends in first order on the values of the two moments of inertia at $I=0$ irrespective of their origin (liquid drop, shell effects, pairing, etc.). Higher order effects, such as changes in the ground state and saddle deformations, and changes

in the shell and pairing effects occur at higher angular momenta. The evidence for the goodness of the cranking approximation and thus for the lack of change of the shell effect in the ground state is evident in the fact that ^{254}No (as well as most strongly deformed rare earth and actinide nuclei) is a good rigid rotor up to $I=20$ [3]. The moment of inertia changes by $\approx 10\%$ from $I=0$ to $I=20$.

The same arguments must hold for the cranking approximation evaluated at the saddle point.

We can test our expectations on the calculations given in the work of Egido *et al.* [2]. In [2] the fission barriers have been calculated at high angular momentum in the cranked Hartree-Fock-Bogoliubov approximation without zero-point energy corrections. The total energies were calculated for different angular momenta as a function of deformation in these complete, microscopic calculations. We can test how well perturbation theory describes these energies by plotting those of the ground state and the first saddle point as a function of $I(I+1)$ (see Fig. 2). We observe that two constant values of the moment of inertia can describe separately the ground state and saddle point energies up to $I=40$.

In typical lighter nuclei, the saddle point is controlled by the liquid drop contributions and is found to be located at large deformations. Therefore, $\mathcal{J}_g \ll \mathcal{J}_s$ and

$$\Delta B \approx \hbar^2 I(I+1)/2\mathcal{J}_g. \quad (3)$$

This produces the large effect of angular momentum on the fission barrier for lighter systems.

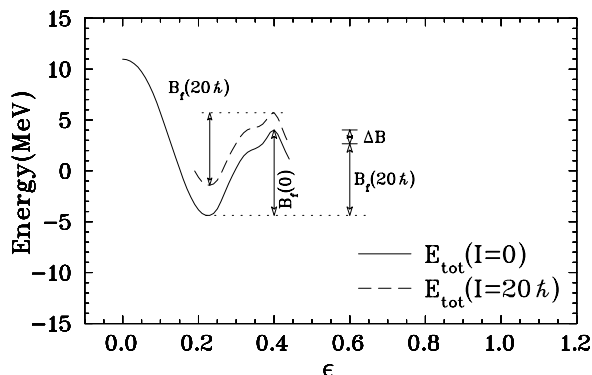


FIG. 1: A schematic description of the fission barrier for ^{254}No . The solid line represents the total energy as a function of deformation when $I = 0$, while the dashed line is calculated for the value of I indicated in the figure.

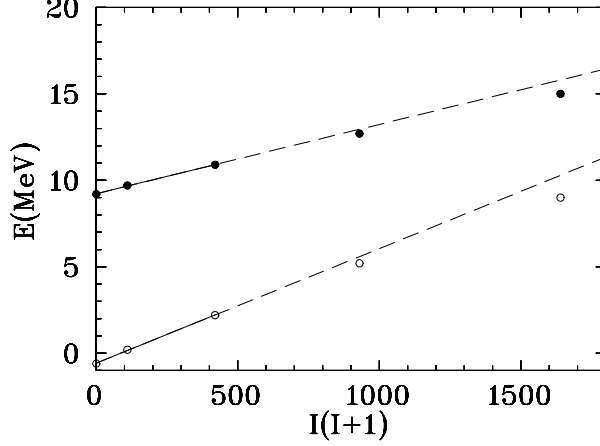


FIG. 2: The total energy of ^{254}No calculated by Egido and Robledo for the ground state (open circles) and saddle (solid circles) are plotted as a function of $I(I+1)$. The solid line is a fit up to $I=20$ while the dashed line is an extrapolation.

However, in trans-Fermium nuclei, the ground state is already deformed at the values of ϵ typical of all actinides. The saddle occurs at a deformation only slightly greater, corresponding to the anti-shell immediately following the deformed minimum. We can rewrite Eq. (2) in terms of the fractional difference of the moments of inertia: small values of $\Delta\mathcal{J}$

$$\Delta\mathcal{J} = \mathcal{J}_s - \mathcal{J}_g, \quad (4)$$

obtaining

$$\Delta B = \frac{\hbar^2 I(I+1)}{2\mathcal{J}_g} \frac{\Delta\mathcal{J}}{\mathcal{J}_s} = E_{rot}^{gs} \frac{\Delta\mathcal{J}}{\mathcal{J}_s}. \quad (5)$$

Consequently, the decrease in barrier height is equal to the ground state rotational energy (E_{rot}^{gs}) times the fractional change in the moment of inertia. For $I=20$, the rotational energy $E_{rot}^{gs} \approx 2.8$ MeV and $\Delta\mathcal{J}/\mathcal{J}_s \approx 0.40$ giving $\Delta B \approx 1.1$ MeV. These features are shown pictorially in Fig. 1.

Furthermore, first order perturbation can be applied to the calculations in [2] and such an approach adequately explains the calculated change in the fission barrier for $I=20$ ($\Delta B \approx 1.1$ MeV).

There is little doubt that the estimated change in the fission barrier based on first order perturbation theory explains the resiliency of ^{254}No against fission for increasing angular momentum, observed both experimentally [1] and in complete microscopic calculations [2]. The same arguments speak for a similar or even greater resilience in superheavy nuclei.

Thus, we expect that we can safely graze in the pastures of the superheavy island of stability, without fear of (moderate) angular momentum values.

Acknowledgments

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 - [3] P. Reiter *et al.*, Phys. Rev. Lett. **82**, 509 (1999).